A New Biased Model Order Reduction for Higher Order Interval Systems

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Abstract. *This paper presents a new biased method for order reduction of linear continuous time interval systems. This method is based on the Stability equation method, Pade approximation and Kharitonov's theorem. The higher order interval system is represented by four Kharitonov transfer functions using the Kharitonov's theorem, and then reduced order models are obtained by the general form of the Stability equation method and Pade approximation. The Stability equation method is used to obtain a reduced order denominator polynomial while the Pade approximation is used for reduced order numerator coefficients. This method generates a stable reduced order model if the original higher order interval system is stable. The proposed method is illustrated with the help of typical numerical examples considered from the literature, and these are compared with well-known methods to show the efficacy of the proposed method.*

Keywords

Interval system, Kharitonov's theorem, model order reduction, Pade approximation, stability equation method.

1. Introduction

In recent decades, to carry out effective research much effort has been made in the field of model order reduction. The original system is of higher order and it is cumbersome. This nature of higher order system analysis is both tedious and costly. The understanding of the behaviour of the system is difficult due to complexity. To avoid the above problems, order reduction implementation is necessary. Model Order Reduction (MOR) is a branch of systems and control theory for reducing the complexity of a higher system while preserving their input - output behaviour. Order reduction methods are broadly classified into two types. Frequency domain order reduction methods are for a transfer function model. Time domain order reduction methods are for a state space model. Several methods are available in the literature for the order reduction of linear continuous systems in the time domain as well as the frequency domain. The reduced order model obtained in the frequency domain gives better matching of the impulse response with its higher order system. Some of the most popularly used frequency domain order reduction methods are Pade approximation and continued fraction methods. These are computationally fast and being able to exactly match the maximum number of system parameters (usually time moments or Markov parameters) to the reduced model. But these methods have a disadvantage - the stability of the reduced model is not guaranteed for a stable system. Efforts have been devoted to developing stability preserving methods such as the Routh stability criterion, Mihailov criterion etc. The stability of these methods is achieved, but the disadvantage of these methods is loss of accuracy. Among these various model order reduction methods for stability preservation available in the literature, the stability equation method [2] is one of the most popular techniques. The advantage of this method is that it preserves stability in the reduced model, if the original high-order system is stable, and retains the first two time–moments of the system. These methods are applicable for fixed - coefficients systems only. However, in many systems the coefficients are constants but uncertain within a finite range. Such systems are classified as interval systems. The above methods are applicable for fixed systems only. In [4] $\gamma - \delta$ Routh Approximation method for interval systems is proposed. The reduced model of interval system is unstable even when the original higher order interval system is stable. An improvement is proposed in [5] to the $\gamma - \delta$ Routh approximation for interval systems using the Kharitonov's polynomials such that the resulting interval Routh approximant is robustly stable. To improve the effectiveness of model order reduction, many mixed methods have been proposed recently [8], [9], [10] based on interval arithmetic. As these methods use interval arithmetic, the stability of the reduced order model is not guaranteed, if the original interval system is stable. In [11] Kharitonov's theorem on robust Hurwitz polynomials is simplified for low-order polynomials. In [12] an interval arithmetic is used to construct a generalized Routh table for determining the denominator polynomial of the reduced system. The reduced numerator polynomial is obtained using the factor division method and the steady state error is minimized using the gain correction factor.

Reduction techniques for linear interval systems using the Kharitonov's theorem are presented [13] in [14] to generate stable reduced technique for linear interval models. In [15] recently a reduction technique for linear interval systems using Kharitonov's theorem interval and Routh-Approximation is presented to generate a stable reduced order interval system. In [16] the reduced order interval model is obtained using Kharitonov's polynomials, which retains stability and full impulse response energy of the higher order interval system in its reduced order interval model.

In this paper, model order reduction of interval systems is carried out by using the Kharitonov's theorem, stability equation method and Pade approximation method. The denominator of the reduced model is obtained by the stability equation method and the numerator is obtained by the Pade approximation method. Thus the stability is guaranteed if the original system is stable.

2. Problem Formulation

Consider a higher order continuous interval system by the transfer function:

$$G_n(s) = \frac{[B_0^-, B_0^+] + [B_1^-, B_1^+]s + \ldots + [B_{n-1}^-, B_{n-1}^+]s^{n-1}}{[A_0^-, A_0^+] + [A_1^-, A_1^+]s + \ldots + [A_n^-, A_n^+]s^n}, \tag{1}$$

where $[A_i^-, A_i^+]$ for $i = 0, 1, \ldots, n$ are denominator coefficients of $G_n(s)$ with A_i^- and A_i^+ as lower and upper bounds of interval $[A_i^-, A_i^+]$ respectively, and $[B_i^-, B_i^+]$ for $i = 0, 1, \ldots, n-1$ are numerator coefficients of $G_n(s)$ with B_i^- and B_i^+ as lower and upper bounds of interval $[B_i^-, B_i^+]$ respectively.

It is proposed to obtain a reduced order interval model of the form:

$$G_r(s) = \frac{[b_0^-, b_0^+] + [b_1^-, b_1^+]s + \ldots + [b_{r-1}^-, b_{r-1}^+]s^{r-1}}{[a_0^-, a_0^+] + [a_1^-, a_1^+]s + \ldots + [a_r^-, a_r^+]s^r}, \tag{2}$$

where $[a_i^-, a_i^+]$ for $i = 0, 1, \ldots, r$ are denominator coefficients of $G_r(s)$ with a_i^- and a_i^+ as lower and upper bounds of interval $[a_i^-, a_i^+]$ respectively, and $[b_i^-, b_i^+]$ for $i = 0, 1, \ldots, r-1$ are numerator coefficients of $G_r(s)$ with b_i^- and b_i^+ as lower and upper bounds of interval $[b_i^-, b_i^+]$ respectively.

3. Proposed Method

Theorem 1 (Kharitonov theorem). *An interval polynomial family $p(s) = \sum_{i=0}^{n}[\alpha_i^-, \alpha_i^+]$ with invariant degree is robustly stable if its four Kharitonov polynomials are stable.*

According to the Kharitonov theorem, every interval polynomial $p(s)$ is associated with four following fixed parameter polynomials called Kharitonov polynomials. They are defined as:

$$\begin{aligned} K_1(s) &= \alpha_0^- + \alpha_1^- s + \alpha_2^+ s^2 + \ldots + \alpha_n^- s^n \\ K_2(s) &= \alpha_0^- + \alpha_1^+ s + \alpha_2^+ s^2 + \ldots + \alpha_n^- s^n \\ K_3(s) &= \alpha_0^+ + \alpha_1^- s + \alpha_2^- s^2 + \ldots + \alpha_n^+ s^n \\ K_4(s) &= \alpha_0^+ + \alpha_1^+ s + \alpha_2^- s^2 + \ldots + \alpha_n^+ s^n \end{aligned}. \tag{3}$$

The interval system is stable if and only if its four Kharitonov polynomials satisfies Routh Hurwitz stability criterion

3.1. Reduction Procedure

Consider a family of real interval transfer function:

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{[B_0^-, B_0^+] + [B_1^-, B_1^+]s + ... + [B_{n-1}^-, B_{n-1}^+]s^{n-1}}{[A_0^-, A_0^+] + [A_1^-, A_1^+]s + ... + [A_n^-, A_n^+]s^n}. \tag{4}$$

The four fixed Kharitonov's transfer functions associated with $G_n(s)$ are given as:

$$\begin{aligned} G_n^1(s) &= \frac{N_n^1(s)}{D_n^1(s)} = \\ &= \frac{B_0^- + B_1^- s + B_2^+ s^2 + \ldots + B_{n-1}^- s^{n-1}}{A_0^- + A_1^- s + A_2^+ s^2 + \ldots + A_n^- s^n} \\ &= \frac{B_{10}^- + B_{11}^- s + B_{12}^+ s^2 + \ldots + B_{1(n-1)}^- s^{n-1}}{A_{10}^- + A_{11}^- s + A_{12}^+ s^2 + \ldots + A_{1n}^- s^n}. \end{aligned} \tag{5}$$

$$G_n^2(s) = \frac{N_n^2(s)}{D_n^2(s)} = \tag{6}$$
$$= \frac{B_0^- + B_1^+ s + B_2^+ s^2 + \ldots + B_{n-1}^- s^{n-1}}{A_0^- + A_1^+ s + A_2^+ s^2 + \ldots + A_n^- s^n}$$
$$= \frac{B_{20}^- + B_{21}^+ s + B_{22}^+ s^2 + \ldots + B_{2(n-1)}^- s^{n-1}}{A_{20}^- + A_{21}^+ s + A_{22}^+ s^2 + \ldots + A_{2n}^- s^n}.$$

$$G_n^3(s) = \frac{N_n^3(s)}{D_n^3(s)} = \tag{7}$$
$$= \frac{B_0^+ + B_1^- s + B_2^- s^2 + \ldots + B_{n-1}^+ s^{n-1}}{A_0^+ + A_1^- s + A_2^- s^2 + \ldots + A_n^+ s^n}$$
$$= \frac{B_{30}^+ + B_{31}^- s + B_{32}^- s^2 + \ldots + B_{3(n-1)}^+ s^{n-1}}{A_{30}^+ + A_{31}^- s + A_{32}^- s^2 + \ldots + A_{3n}^+ s^n}.$$

$$G_n^4(s) = \frac{N_n^4(s)}{D_n^4(s)} = \tag{8}$$
$$= \frac{B_0^+ + B_1^+ s + B_2^- s^2 + \ldots + B_{n-1}^+ s^{n-1}}{A_0^+ + A_1^+ s + A_2^- s^2 + \ldots + A_n^+ s^n}$$
$$= \frac{B_{40}^+ + B_{41}^+ s + B_{42}^- s^2 + \ldots + B_{4(n-1)}^+ s^{n-1}}{A_{40}^+ + A_{41}^+ s + A_{42}^- s^2 + \ldots + A_{4n}^+ s^n}.$$

The above Kharitonov's transfer functions are, in general, represented as:

$$G_n^I(s) = \frac{N_n^I(s)}{D_n^I(s)} = \frac{\sum_{j=0}^{n-1} B_{Ij} s^j}{\sum_{j=0}^{n} A_{Ij} s^j}. \tag{9}$$

1) Step 1

Determination of the denominator coefficients of lower order system for first Kharitonov transfer function by stability equation method:

For I=1:

$$G_n^1(s) = \frac{N_n^1(s)}{D_n^1(s)} = \frac{B_{10}^- + B_{11}^- s + B_{12}^+ s^2 + \ldots + B_{1(n-1)}^- s^{n-1}}{A_{10}^- + A_{11}^- s + A_{12}^+ s^2 + \ldots + A_{1n}^- s^n}. \tag{10}$$

For first Kharitonov transfer function the reduced order model is:

$$G_r^1(s) = \frac{N_r^1(s)}{D_r^1(s)} = \frac{B_{10}^- + B_{11}^- s + B_{12}^+ s^2 + \ldots + B_{1(r-1)}^- s^{r-1}}{A_{10}^- + A_{11}^- s + A_{12}^+ s^2 + \ldots + A_{1r}^- s^r}. \tag{11}$$

For stable first Kharitonov transfer function G_n^1 (s), the denominator $D_n^1(s)$ of the Higher Order System (HOS) is bifurcated in the even and odd parts in the form of stability equations as:

$$\left.\begin{aligned} D_e^n(s) &= A_{10} \prod_{i=1}^{m_1} \left(1 + \frac{s^2}{z_i^2}\right) \\ D_o^n(s) &= A_{11} \prod_{i=1}^{m_2} \left(1 + \frac{s^2}{p_i^2}\right) \end{aligned}\right\}, \tag{12}$$

where m_1 and m_2 are the integer parts of $n/2$ and $(n-1)/2$ respectively and $z_1^2 < p_1^2 < z_2^2 < p_2^2$.

Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in Eq. (12), the stability equations for r^{th} order Lower Order System (LOS) are obtained as:

$$\left.\begin{aligned} D_e^r(s) &= a_{10} \prod_{i=1}^{m_3} \left(1 + \frac{s^2}{z_i^2}\right) \\ D_o^r(s) &= a_{11} \prod_{i=1}^{m_4} \left(1 + \frac{s^2}{p_i^2}\right) \end{aligned}\right\}, \tag{13}$$

where m_3 and m_4 are the integer parts of $r/2$ and $(r-1)/2$, respectively.

Combining these reduced stability equations and therefore properly normalizing it, the r^{th} order denominator of Lower Order System (LOS) is obtained as:

$$D_r^1(s) = D_e^r(s) + D_o^r(s) = \sum_{i=0}^{r} a_{1i} s^i. \tag{14}$$

Therefore, the denominator polynomial in Eq. (11) is now known, which is given by:

$$\begin{aligned} D_r^1(s) = a_{10} + a_{11}s + a_{12}s^2 + \ldots + \\ + a_{1(r-1)}s^{r-1} + a_{1r}s^r. \end{aligned} \tag{15}$$

The same procedure is employed for remaining Kharitonov's transfer function for reduced order denominator.

$$\begin{aligned} D_r^2(s) = a_{20} + a_{21}s + a_{22}s^2 + \ldots + \\ + a_{2(r-1)}s^{r-1} + a_{2r}s^r, \\ D_r^3(s) = a_{30} + a_{31}s + a_{32}s^2 + \ldots + \\ + a_{3(r-1)}s^{r-1} + a_{3r}s^r, \\ D_r^4(s) = a_{40} + a_{41}s + a_{42}s^2 + \ldots + \\ + a_{4(r-1)}s^{r-1} + a_{4r}s^r. \end{aligned} \tag{16}$$

2) Step 2

Determination of the numerator coefficients of the reduced model by Pade approximation:

For the first Kharitonov transfer function, it can be expanded in the power series about s=0 as:

$$\begin{aligned} G_n^1(s) &= \frac{\sum\limits_{i=0}^{n-1} B_{1i}s^i}{\sum\limits_{i=0}^{n} A_{1i}s^i} = \\ &= E_{10} + E_{11}s + E_{21}s^2 + \ldots \end{aligned} \tag{17}$$

The coefficients of the power series expansion can also be calculated as follows:

$$E_{10} = \frac{B_{10}}{A_{10}}. \tag{18}$$

$$\begin{aligned} E_{1i} &= \frac{1}{A_{10}}\left[B_{1i} - \sum_{j=1}^{i} A_{1j}E_{1(i-j)}\right] & i > 0, \\ B_{1i} &= 0 & i > n-1. \end{aligned} \tag{19}$$

The r^{th} order reduced model for the first Kharitonov transfer function is taken as:

$$G_r^1(s) = \frac{N_r^1(s)}{D_r^1(s)} = \frac{\sum\limits_{i=0}^{r-1} b_{1i}s^i}{\sum\limits_{i=0}^{r} a_{1i}s^i}. \tag{20}$$

Here $D_r^1(s)$ is known through the stability equation method.

For $N_r^1(s)$ of equation to be Pade approximants of $G_n^1(s)$ of equation, we have:

$$\left.\begin{aligned} &b_{10} = a_{10}E_{10} \\ &b_{11} = a_{10}E_{11} + a_{11}E_{10} \\ &\vdots \\ &b_{r-1} = a_{10}E_{1(r-1)} + \ldots + a_{r-2}E_{11} + a_{r-1}E_{10} \end{aligned}\right\}, \tag{21}$$

the coefficients $b_{1j}; j = 0, 1, 2, \ldots, r-1$ can be found by solving the above r linear equations.

Hence the numerator $N_r^1(s)$ is obtained as:

$$N_r^1(s) = b_{10} + b_{11}s + b_{12}s^2 + \ldots + b_{r-1}s^{r-1}. \tag{22}$$

The **Step 1** and **Step 2** procedure is repeated for remaining Kharitonov transfer function.

After obtaining four Kharitonov reduced order transfer functions, the reduced order interval transfer function is obtained by using the below equation:

$$G_r(s) = \frac{\sum\limits_{j=0}^{r-1} \left[\min(b_{Ij}), \max(b_{Ij})\right] s^j}{\sum\limits_{j=0}^{r} \left[\min(a_{Ij}), \max(a_{Ij})\right] s^j}. \tag{23}$$

Integral Squared Error (ISE):

The Integral Squared Error (ISE) between the original higher order system and reduced order system is represented in the form:

$$j = \int_0^\infty \left[y(t) - y_r(t)\right]^2 \mathrm{d}t. \tag{24}$$

Mathematically, the integral squared error can be represented as:

$$j = \sum_{i=0}^{N} \left[y(t) - y_r(t)\right]^2, \tag{25}$$

where, *y(t)* is the unit step response of higher order and $y_r(t)$ is the unit step response lower order system at the t^{th} instant in the time interval $0 \leq t \leq N$, where N is to be chosen.

3.2. Numerical Example

Example 1. *Consider a higher order interval system* [4]:

$$\begin{aligned} &G(s) = \\ &= \frac{[2,3]s^2 + [17.5, 18.5]s + [15, 16]}{[2,3]s^3 + [17,18]s^2 + [35,36]s + [20.5, 21.5]}. \end{aligned} \tag{26}$$

This higher order interval system can be represented as four Kharitonov higher order transfer functions given as:

$$\begin{aligned} G_3^1(s) &= \frac{3s^2 + 17.5s + 15}{3s^3 + 18s^2 + 35s + 20.5}, \\ G_3^2(s) &= \frac{3s^2 + 18.5s + 15}{2s^3 + 18s^2 + 36s + 20.5}, \\ G_3^3(s) &= \frac{2s^2 + 17.5s + 16}{3s^3 + 17s^2 + 35s + 21.5}, \\ G_3^4(s) &= \frac{2s^2 + 18.5s + 16}{2s^3 + 17s^2 + 36s + 21.5}. \end{aligned} \tag{27}$$

From the first Kharitonov transfer function:

$$G_3^1(s) = \frac{3s^2 + 17.5s + 15}{3s^3 + 18s^2 + 35s + 20.5}. \tag{28}$$

Step 1: Bifurcating the denominator of the above Higher Order System (HOS) in even and odd parts, we get the stability equations as:

$$D_e^n(s) = 20.5 + 18s^2 = 20.5\left[1 + \frac{s^2}{1.14}\right], \tag{29}$$

$$D_0^n(s) = 35s + 3s^3 = 35s\left[1 + \frac{s^2}{11.67}\right]. \tag{30}$$

Now by discarding the factors with large magnitude of z_i^2 and p_i^2 in $D_e^n(s)$and $D_o^n(s)$ respectively, the stability equations for the second-order reduced model are given by:

$$\begin{aligned}
D_e^r(s) &= 20.5\left[1+\frac{s^2}{1.14}\right],\\
D_0^r(s) &= 35s,\\
D_r^1 &= D_e^r(s)+D_o^r(s),\\
&= 20.5+17.99s^2+35s,\\
&= s^2+1.94s+1.14.
\end{aligned} \tag{31}$$

The reduced model:

$$G_r^1(s) = \frac{b_{11}s+b_{10}}{s^2+1.94s+1.14}. \tag{32}$$

Step 2: By using Pade approximation method For the first Kharitonov transfer function:

$$\begin{aligned}
G_3^1(s) &= \frac{B_{12}s^2+B_{11}s+B_{10}}{A_{13}s^3+A_{12}s^2+A_{11}s+A_{10}},\\
&= \frac{3s^2+17.5s+15}{3s^3+18s^2+35s+20.5}.
\end{aligned} \tag{33}$$

The reduced denominator obtained by **Step 1**:

$$G_r^1(s) = \frac{b_{11}s+b_{10}}{a_{12}s^2+a_{11}s+a_{10}} = \frac{b_{11}s+b_{10}}{s^2+1.94s+1.14}, \tag{34}$$

$$E_{1i} = \frac{1}{A_{10}}\left[B_{1i}-\sum_{j=1}^{i}A_{1j}E_{1(i-j)}\right], \tag{35}$$

$$e_{1i} = \frac{1}{a_{10}}\left[b_{1i}-\sum_{j=1}^{i}a_{1j}e_{1(i-j)}\right], \tag{36}$$

$$E_{1i} = e_{1i}. \tag{37}$$

Substituting i=0 in Eq. (35) and Eq. (36):

$$\frac{B_{10}}{A_{10}} = \frac{b_{10}}{a_{10}}, \tag{38}$$

$$\frac{15}{20.5} = \frac{b_{10}}{1.14}, \tag{39}$$

$$b_{10} = 0.84. \tag{40}$$

Substituting i=1 in Eq. (35), Eq. (36) and Eq. (37):

$$b_{11} = 0.97222. \tag{41}$$

Then reduced numerator is:

$$N_r^1(s) = 0.97s+0.84. \tag{42}$$

The reduced order of first Kharitonov transfer function is:

$$G_2^1(s) = \frac{0.97s+0.84}{s^2+1.94s+1.14}. \tag{43}$$

Same as for remaining Kharitonov transfer function the reduced order transfer functions are:

$$G_2^2(s) = \frac{1.03s+0.84}{s^2+2s+1.14}, \tag{44}$$

$$G_2^3(s) = \frac{1.03s+0.94}{s^2+2.06s+1.26}, \tag{45}$$

$$G_2^4(s) = \frac{1.08s+0.94}{s^2+2.12s+1.26}. \tag{46}$$

Therefore, finally reduced order interval model obtained by using Eq. (23):

$$G_2(s) = \frac{[0.97,1.08]s+[0.84,0.94]}{[1,1]s^2+[1.94,2.12]s+[1.14,1.26]}. \tag{47}$$

Comparing this with model order reduction using Mihailov criterion and Cauer Second form in Eq. (48) [10].

Proof for Stability

According to [11], the necessary and sufficient condition for robust stability of interval polynomial for order $n=1$ and $n=2$ is positive lower bounds on the coefficients. The denominator of Reduced Order Interval Model (ROIM) obtained by the proposed method is:

$$D_2(s) = [1,1]s^2+[1.94,2.12]s+[1.14,1.26]. \tag{49}$$

It is seen from the denominator of ROIM from the Exm. 1 that the lower bound coefficients are positive, hence it can be stated that the system is robustly stable.

The step response of the proposed reduced interval system compared to [10].

Tab. 1: Comparison of integral square error for reduced interval system model.

Method of order reduction	**ISE for lower limit**	**ISE for upper limit**
Proposed Method	0.0019785	0.00174763
(D.Kranthi Kumar, S.K.Nagar and J.P.Tiwari, October 2011)[10]	0.0089	0.0113

It has been observed from Fig. 1 and Fig. 2 the step responses of the lower and upper bounds of the original higher order interval system and the reduced order interval model obtained by the proposed method are closely matching. And thereby the reduced order interval model retains the stability. It is observed that the time moments of the original system and model obtained by the proposed reduced order system matches, and also has better matching of transient and steady state response than [10]. It is also observed from Tab. 1 that the ISEs of the lower and upper bounds of the reduced order transfer functions model obtained by the proposed method are lesser than the method given in [10]. As the method in [10] uses the interval arithmetic for order reduction, thereby sometimes generates unstable reduced order interval models for stable higher order interval systems. Whereas the proposed method generate stable models by the use of Kharitonov's theorem. Hence the proposed method avoids the difficulty of generating unstable models.

$$R_2(s) = \frac{[11.1949, 20.3706]s + [14.1674, 16.9413]}{[17.0011, 18.0007]s^2 + [31.3826, 33.6111]s + [20.3061, 21.2052]}. \quad (48)$$

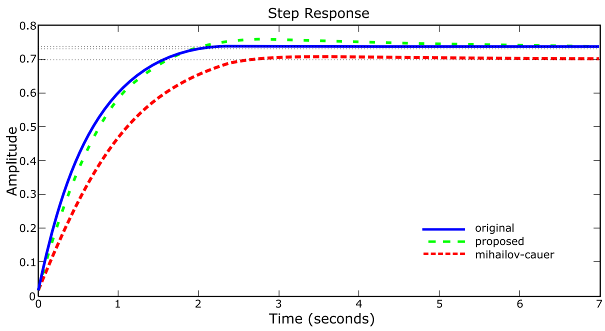

Fig. 1: Step response comparison of lower bounds.

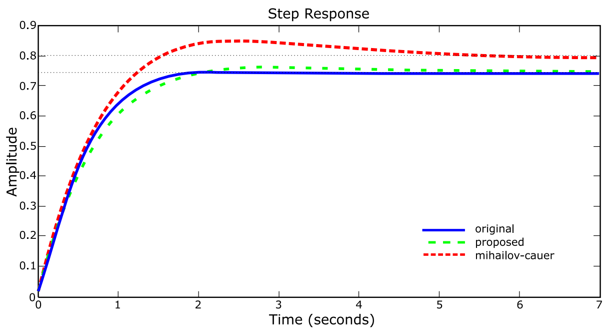

Fig. 2: Step response comparison of upper bounds.

Example 2. *Consider a higher order interval system* [12]:

$$\begin{aligned} G_n(s) &= \\ &= \frac{[1.9, 2.1]s^6 + [24.7, 27.3]s^5 +}{[0.95, 1.05]s^7 + [8.779, 9.703]s^6 +} \cdots \\ &\cdots \frac{+[157.7, 174.3]s^4 +}{+[52.23, 57.73]s^5 + [182.9, 202.1]s^4 +} \cdots \\ &\cdots \frac{+[542, 599]s^3 + [930, 1028]s^2 +}{+[429.1, 474.2]s^3 + [572.632.7]s^2 +} \cdots \\ &\cdots \frac{+[721.8, 797.8]s + [187.1, 206.7]}{+[325.3, 359.5]s + [57.35, 63.39]}. \end{aligned} \quad (50)$$

This higher order interval system can be represented as four Kharitonov higher order transfer functions given as:

$$\begin{aligned} G_7^1(s) &= \\ &= \frac{2.1s^6 + 24.7s^5 + 157.5s^4 + 599s^3 +}{1.05s^7 + 9.703s^6 + 52.23s^5 + 182.9s^4 +} \cdots \\ &\cdots \frac{+1028s^2 + 721.8s + 187.1}{+474.2s^3 + 632.7s^2 + 325.3s + 57.35}. \end{aligned} \quad (51)$$

$$\begin{aligned} G_7^2(s) &= \\ &\frac{2.1s^6 + 27.3s^5 + 157.5s^4 + 542s^3 +}{0.95s^7 + 9.703s^6 + 57.73s^5 + 182.9s^4 +} \cdots \\ &\cdots \frac{+1028s^2 + 797.8s + 187.1}{+429.1s^3 + 632.7s^2 + 359.5s + 57.35}. \end{aligned} \quad (52)$$

$$\begin{aligned} G_7^3(s) &= \\ &\frac{1.9s^6 + 24.7s^5 + 174.3s^4 + 599s^3 +}{1.045s^7 + 8.779s^6 + 52.23s^5 + 202.1s^4 +} \cdots \\ &\cdots \frac{+930s^2 + 721.8s + 206.7}{+474.2s^3 + 474.2s^2 + 325.3s + 63.39}. \end{aligned} \quad (53)$$

$$\begin{aligned} G_7^4(s) &= \\ &\frac{1.9s^6 + 27.3s^5 + 174.3s^4 + 542s^3 +}{1.05s^7 + 8.779s^6 + 57.7s^5 + 202.1s^4 +} \cdots \\ &\cdots \frac{+930s^2 + 797.8s + 206.7}{+429s^3 + 572.5s^2 + 359.5s + 63.39}. \end{aligned} \quad (54)$$

The reduced order of four Kharitonov transfer functions obtained by using **Step 1**, **Step 2** in Proposed Method described in section 3. is as follows:

$$G_2^1(s) = \frac{721.8s + 187.1}{615.75s^2 + 325.3s + 57.35}, \quad (55)$$

$$G_2^2(s) = \frac{797.8s + 187.1}{615.75s^2 + 359.50s + 57.35}, \quad (56)$$

$$G_2^3(s) = \frac{721.8s + 206.7}{549.29s^2 + 325.30s + 63.39}, \quad (57)$$

$$G_2^4(s) = \frac{797.8s + 206.7}{549.29s^2 + 359.50s + 63.39}. \quad (58)$$

Therefore, finally reduced order interval model obtained by using Eq. (23), in Eq. (59):

$$\begin{aligned} G_2(s) &= \frac{[721.8, 797.8]s +}{[549.29, 615.75]s^2 + [325.3, 359.5]s +} \cdots \\ &\cdots \frac{+[187.1, 206.7]}{+[57.35, 63.39]}. \end{aligned} \quad (59)$$

The reduced order interval model is compared with well-known methods in literature. The reduced order interval model obtained by using a method from [4]:

$$R_{2b}(s) = \frac{[1.61, 1.84]s + [0.27, 0.53]}{[1, 1]s^2 + [0.52, 0.83]s + [0.08, 0.16]}. \quad (60)$$

The reduced order interval model obtained by using a method from [12]:

$$\begin{aligned} R_{2s}(s) &= \\ &\frac{[260.955, 861.331]s + [175.232, 218.581]}{[364.72, 366.62]s^2 + [281.08, 282.35]s + [59.74, 61]}. \end{aligned} \quad (61)$$

Proof for Stability

According to [11], the necessary and sufficient condition for robust stability of interval polynomial for order $n = 1$ and $n = 2$ is positive lower bounds on the coefficients. The denominator of Reduced Order Interval Model (ROIM) obtained by the proposed method is:

$$D_2(s) = [549.29, 615.75]s^2 + [325.3, 359.5]s + \cdots + [57.35, 63.39]. \tag{62}$$

It is seen from the denominator of ROIM from the Exm. 1 that the lower bound coefficients are positive, hence it can be stated that the system is robustly stable.

Tab. 2: Comparison of integral squared error for reduced interval system model.

Method of order reduction	ISE for lower limit	ISE for upper limit
Proposed Method	0.8989	0.2029
(Bandyopadhyay) [4]	2.2599	5.954
(Selvaganesan) [12]	3.0105	4.0216

The step response of the proposed reduced interval system compared to [4], [12].

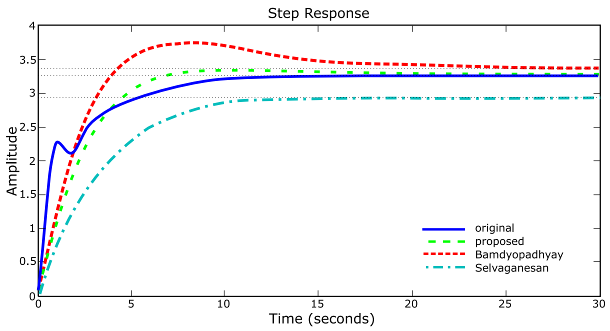

Fig. 3: Step response comparison of lower bounds.

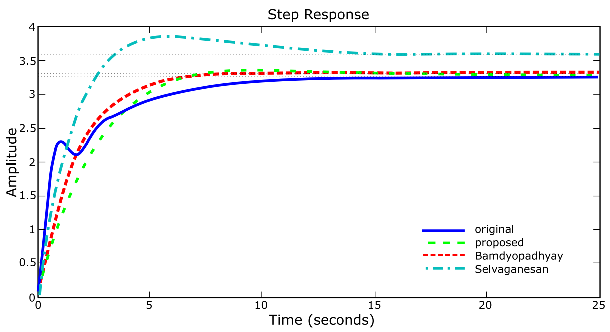

Fig. 4: Step response comparison of upper bounds.

It has been observed from Fig. 3 and Fig. 4 the step responses of the lower and upper bounds of the original higher order interval system and the reduced order interval model obtained by the proposed method are closely matching. And thereby the reduced order interval model retains the stability. It is observed that the time moments of the original system and model obtained by the proposed reduced order system matches, and also has better matching of transient and steady state response than [4] and [12]. It is also observed from Tab. 2 that the ISEs of the lower and upper bounds of the reduced order transfer functions model obtained by the proposed method are lesser than the methods given in [4] and [12]. The methods given in [4] and [12] use the interval arithmetic for order reduction, thereby sometimes generate unstable reduced order interval models for stable higher order interval systems. Whereas the proposed method uses the Kharitonov's theorem for order reduction; hence it avoids the difficulty in generating unstable models.

4. Conclusion

In this paper, a biased method of order reduction is proposed. The reduced model of denominator polynomial is obtained by using the stability equation method and the numerator is determined by the Pade approximation. It has been observed that the time moments of the reduced order model obtained by the proposed method matches with the original system. In [4], [10], and [12] the reduced order interval model is obtained by using interval arithmetic, hence the use of interval arithmetic, sometimes generating unstable reduced order model for stable higher order interval system. The proposed method guarantees the stability of reduced order model if the original system is stable and minimizes the ISE. The response of the reduced model is good.

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